Making Beehives

Driving Question

Which 3-dimensional shapes do bees use to make their beehives?

Background

Bees instinctively understand the need for efficiency when building their hives. They want to construct their nests so that they achieve a maximum amount of enclosed volume with a minimum of material and effort needed to build them. Bees could be considered to be one of nature's logistical engineers.

Logistics engineers apply mathematical and scientific principles of engineering to the distribution of services and goods that people need on a daily basis. They do this to optimize the infrastructure that provides us with the things we need. Logistics engineers work with the systems and processes related to distributing goods and services, often called the supply chain. They are interested in making the distribution of goods and services as efficient as possible because this helps to improve profits.

If you have ever had the chance to observe a container ship loaded with shipping containers, you have seen the results of a logistics engineer's work. The container ship is perfectly designed to carry the maximum load safely and as rapidly as possible from one port (a point of distribution) to another. You can imagine how the cost of goods would be impacted if those goods were transported across an ocean scattered across the deck of a sailing ship that depended on the wind for power. Logistics engineers, like bees building their hives, employ knowledge of math and science to produce the most efficient structures or processes possible with the materials available.

Project Objectives

In this project you will work with the members of your group to:

Mandatory

- Understand which polygons can be used to tessellate a plane, and extend this reasoning from two dimensions to three dimensions.
- Set areas of a square, circle, equilateral triangle, and hexagon equal to each other for comparison of the ratio of perimeter to area.
- Determine the polygon that would most economically hold the greatest volume of liquid based on comparison of base area. Compute the theoretical volume.
- Using available materials construct a set prisms and a cylinder of whose bases are of equal area, and whose heights are equal.
- Determine experimentally the volume of each prism and cylinder using liquid, and compare these results to the theoretical values previously computed.
- Relate bees' food energy needs to their optimization of resources used to build honeycombs.

Optional

- Derive the formula for the area of a regular hexagon from the formula for the area of an equilateral triangle.
- Compute and compare the area of a circle to the area of a hexagon inscribed in it, and relate these areas to the volume of the cylinder and hexagonal prism.

- Compute and compare the area of the annulus formed by circles inscribed in and circumscribed about a hexagon, to the area of the hexagon.
- Describe and explain which transformations (rotation, translation, reflection, dilation) can be applied to produce a hexagonal tessellation.

Materials and Equipment

Mandatory Equipment

- Water-resistant material for making shapes (such as plastic-coated card stock)
- Scissors
- Tape
- Water
- Ruler or straightedge
- Compass
- Drawing paper, several sheets

Optional Equipment

- Graduated cylinder
- Food coloring
- Pattern Blocks
- Dynamic geometry software

Key Concepts for Background Research

Research your project topic based on the driving question above. Use any resources available to research background information that will help you to complete your project.

Below is a list of key concepts that may be helpful when doing your background research.

- Regular polygon
- Polyhedron (plural - polyhedra)
- Non-polyhedron
- Geometric solids
- Volume of solids
- Surface area of polyhedra
- Tessellation
- Prism
- Cylinder
- Platonic solids
- Area of polygons
- Perimeter

Safety and Maintenance

Add this important safety precaution to your normal classroom procedures:

- A construction project involves the use of sharp objects. Handle all sharp objects carefully, including drawing compasses, scissors or craft/hobby knives.

Investigation

Use these guiding questions to help answer the driving question:

PART 1:

1. Which regular polygons can be used to tessellate, or tile, a plane, without overlapping or leaving gaps between them?

2. What three-dimensional solids can be formed from these polygons? How is a cylinder different from these three-dimensional solids?

3. Consider several types of three-dimensional solids, including hexagonal prisms, and cylinders. How can you compare the volume of one solid to another, both theoretically and experimentally?

4. Suppose you want to construct several two-dimensional figures that all have the same area, say 10 cm^2 for example. How can you use the area formulas for a quadrilateral, an equilateral triangle, and a circle to determine the length of each side (or circumference) of the figures?

5. What are the lengths of each side of a square and an equilateral triangle whose areas are both 10 cm^2? What is the length of the diameter of a circle whose area is 10 cm^2?

6. What is the length of each side of a regular hexagon whose area is 10 cm^2?

7. Of all the two-dimensional figures whose areas and lengths of sides you computed, which figures, if any, have shorter perimeters? Is there a "shortest perimeter" figure that can also tessellate a plane?

8. Imagine you need to construct a set of hollow geometric solids, using the least amount of material possible while still holding the largest volume of liquid possible. This is the same task that bees undertake when they construct their honeycombs from beeswax. Mathematically, which geometric solids are best suited for this task?

9. Biologists have shown that a honeybee needs to eat between six and eight grams of honey in order to produce one gram of beeswax. In terms of bees' food energy requirements, discuss why it is important for bees to use beeswax as efficiently as possible. Why do bees build their honeycombs using the shapes they do?

PART 2: (optional)

1. How can the formula for the area of a regular hexagon be derived from the formula for the area of an equilateral triangle?

2. Suppose a certain species of honeybee made its honeycomb with circular cells packed as closely together as possible (hexagonal packing). What volume of beeswax would be necessary to fill in all the gaps between the one cell and the other six cells surrounding it? Since you don't know the size of this species of bee (and therefore the size of its honeycomb cells), express this volume in terms of the ratio of the area of a circle to the area of a hexagon inscribed in that circle.

PART 3: (optional)

1. Consider a hexagon that has a circle inscribed in it and a second circle circumscribed about it. The annulus formed by the two concentric circles has an area A, while the hexagon has an area A'. What percent of A' is A?

2. Of the transformations mathematically possible, which could be applied to the hexagonal honeycomb pattern of beehives?

Synthesis Questions

The answers to the following questions will guide you to further critical thinking about your project topic and will build knowledge to help answer questions your classmates may ask when you make your presentation.

1. Why do only certain kinds of polygons tessellate?

2. What are some other examples of how hexagons are used in nature?

3. How are hexagons used by people in daily life? Why are hexagons chosen as the shape in these examples?

Extension and Real Life Application

The answers to the following questions will help you make connections between the concepts relevant to your project topic and their applications to real world problems, and will further add to your knowledge to help answer questions that your classmates may ask when you present.

1. Discuss at least one real life application (and extension) of concepts relevant to this entire project, in addition to what you have discussed in the previous question(s).

2. Scientists estimate that one-third of food consumed by humans is dependent on insect pollination. Bees carry out the majority of this pollination. Discuss the implications for the world food supply if bees suffer some kind of crisis. Is there any such crisis currently affecting bees?

3. What is "closest packing" and how does it relate to the chemical structures of various crystalline substances?

Mathematics Investigation Report – Assessment Rubric **Level II/III**		**Name:** **Math Class:**				
Component	To receive highest marks the student:	**4** **Expert**	**3** **Prac-titioner**	**2** **Appren-tice**	**1** **Novice**	**0** **No Attempt**
1. Preparation and Research	□ Clearly invested several hours doing relevant research □ Brought and prepared all items necessary for their presentation □ Prepared supporting handout for their presentation □ Prepared a thorough presentation discussion for their classmates					
2. Demonstrations, Models, or Experiments	□ Obtained all necessary materials and used them to thoroughly explore the guiding questions for Part 1 of the investigation □ Completed and documented results, data, and observations of any guiding questions investigated in Parts 1 – 3 of the investigation □ Performed experiments or demonstrations proficiently for others, or explained clearly a model and the concepts the student investigated with the model					
3. Content	□ Presented written and spoken explanations that were mathematically accurate and paraphrased in the student's own words □ Answered all questions posed by their teacher or classmates correctly and thoughtfully □ Answered project synthesis and real-world application questions correctly and thoroughly □ Communicated clearly an understanding of the connection between their model or experiments and the driving question and theory behind the over-arching concept					
4. Technology	□ Prepared a **website** explaining their project and the driving question associated with it, including: • A thorough, well-written explanation of the answer to the driving question and the theory behind the over-arching concept • Supporting pictures, videos, and research references (including web-links) □ Recorded a supporting **video** for the website • Edited the supporting video in an attempt to produce a good–excellent quality video • Included footage of their experimental setup with an explanation • Included a thorough, well-spoken explanation of the answer to the driving question and the theory behind the over-arching concept □ Produced a project **brochure** with 2 **QR codes** linking to video and to website					
5. Interdisciplinary Connections	□ Chose and completed ELA option to best illustrate the project's objectives and monitor student progress □ Included the ELA component on project website as separate page with appropriate design □ Chose a related SS connection based on consultation with SS teacher □ Included the SS connection on project website as separate page with appropriate design □ Included ELA component and SS connection in video presentation					
6. Real-World Application and Extension	□ Identified in written and spoken explanations the application of the topic to the real world, including specific examples □ Thoroughly discussed relevance of the topic to real life					
7. Presentation	□ Delivered their content clearly and thoroughly, in an organized, logical manner □ Integrated their research and experimental setup into the presentation as visual support					
					Total Points for Investigation (Maximum of 24 Points)	

Guidelines for Marks:

4 = **Expert:** Distinguished command of the topic; students show insightful and sophisticated communication of their understanding

3 = **Practitioner:** Strong command of the topic; students show reasonable and purposeful communication of their understanding

2 = **Apprentice:** Moderate command of the topic; students show adequate but basic communication of their understanding

1 = **Novice:** Partial command of the topic; students show limited and insufficient communication of their understanding

0 = **No Attempt**